SINGLE OPTION CORRECT

1. If $x^a y^b = e^m$, $x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are respectively.

(A) $\frac{\Delta_1}{\Delta_3}$ and $\frac{\Delta_2}{\Delta_3}$ (B) $\frac{\Delta_2}{\Delta_1}$ and $\frac{\Delta_3}{\Delta_1}$ (C) $\log\left(\frac{\Delta_1}{\Delta_3}\right)$ and $\log\left(\frac{\Delta_2}{\Delta_3}\right)$ (D) $e^{\frac{\Delta_1}{\Delta_3}}$ and $e^{\frac{\Delta_2}{\Delta_3}}$

2. The system of equations

$$\left.\begin{array}{l} \alpha x + y + z = \alpha - 1 \\ x + \alpha y + z = \alpha - 1 \\ x + y + \alpha z = \alpha - 1 \end{array}\right\}$$, has no solution, if α is

(A) 1 (B) not - 2 (C) either – 2 or 1 (D) - 2

3. Let $\omega = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$, then the value of determinant $\begin{vmatrix} 1+\omega & \omega^2 & -\omega \\ 1+\omega^2 & \omega & -\omega^2 \\ \omega^2+\omega & \omega & -\omega^2 \end{vmatrix}$ is

(A) -3 (B) $-3\omega^2$ (C) $3\omega^2$ (D) 3

4. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -3 \\ 2 & 1 & 0 \end{bmatrix}$ and B = adj(A) and C = 5A, then $\frac{|\text{adj}\, B|}{|C|}$ is equal to

(A) 5 (B) 25 (C) -1 (D) 1

5. If P is a 3×3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3×3 identity matrix, then there exist a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that

(A) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (B) PX = X (C) PX = 2X (D) PX = - X

6. The number of different non-singular matrices of the type $A = \begin{bmatrix} 1 & a & c \\ 1 & 1 & b \\ 0 & -w & w \end{bmatrix}$ where $w = e^{i\theta}$ and

$a, b, c \in \{z : z^4 - 1 = 0\}$ are

(A) 44 (B) 48 (C) 56 (D) 55

7. If adj(B) = A, |P| = |Q| = 1, then $\text{adj}(Q^{-1} B P^{-1})$ is:

(A) PQ (B) QAP (C) PAQ (D) $PA^{-1}Q$

8. Let P = $[a_{ij}]$ be a 3×3 matrix and let Q = $[b_{ij}]$ where $b_{ij} = 2^{i+j} a_{ij}$ for $1 \le i, j \le 3$. If the determinant of P is 2, then the determinant of matrix Q is

(A) 2^{10} (B) 2^{11} (C) 2^{12} (D) 2^{13}

9. How many 3×3 matrices M with entries from {0, 1, 2} are there, for which the sum of the diagonal entries of $M^T M$ is 5?

(A) 126 (B) 198 (C) 162 (D) 135

10. Let matrix $A = \begin{bmatrix} x & y & -z \\ 1 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$, where x, y, z ∈ N. If $\left|\text{adj}\left(\text{adj}\left(\text{adj}\left(\text{adj}A\right)\right)\right)\right| = 4^8.5^{16}$, then the number of such matrices are ?

(A) 28 (B) 45 (C) 36 (D) 55

11. If $Z = \begin{vmatrix} 3+2i & 1 & i \\ 2 & 3-2i & 1+i \\ 1-i & -i & 3 \end{vmatrix}$ and $|z+\bar{z}| = k|z|$, then 3k is

(A) 0 (B) 6 (C) 3 (D) 9

12. If $\begin{vmatrix} x & 3 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix} = Ax^4 + Bx^3 + Cx^2 + Dx + E$, then the value of 5A + 4B + 3C + 2D + E is equal to ?

(A) 0 (B) 15 (C) -16 (D) -17

13. If $A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$, then the value of $\left|A + A^2B^2 + A^3 + A^4B^4 + ..100\,\text{terms}\right|$ is equal to

(A) 1000 (B) - 800 (C) 0 (D) - 8000

14. If $P = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$, $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $Q = PAP^T$ and $X = P^T Q^{2005} P$, then X is equal to

(A) $\begin{bmatrix} 1 & 2005 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 4+2005\sqrt{3} & 6015 \\ 2005 & 4-2005\sqrt{3} \end{bmatrix}$

(C) $\frac{1}{4}\begin{bmatrix} 2+\sqrt{3} & 1 \\ -1 & 2-\sqrt{3} \end{bmatrix}$ (D) $\frac{1}{4}\begin{bmatrix} 2005 & 2-\sqrt{3} \\ 2+\sqrt{3} & 2005 \end{bmatrix}$

15. Let A be a 2×3 matrix whereas B be a 3×2 matrix. If det (AB) = 4, then value of det (BA), is?

(A) 1 (B) -1 (C) 0 (D) 5

16. Evaluate: $\left|\left|A^T\right|A\right|A\right|$ where $A = \begin{bmatrix} 1 & 7 & -1 \\ 2 & 3 & 2 \\ 3 & -1 & 4 \end{bmatrix}$.

(A) 11^{13} (B) 13^{11} (C) 11^{9} (D) None of these

17. A and B are matrices of order 3×2 and 2×3 respectively. If $AB=\begin{bmatrix}8&2&-2\\2&5&4\\-2&4&5\end{bmatrix}$ then

(A) $BA=\begin{bmatrix}0&9\\9&0\end{bmatrix}$ (B) $BA=\begin{bmatrix}9&0\\0&9\end{bmatrix}$ (C) ABAB = 9AB (D) ABAB = AB

MULTI OPTION(S) CORRECT

18. If A and B are 3×3 matrices and $|A| \neq 0$, then which of the following are true?

(A) $|AB| = 0 \rightarrow |B| = 0$
(B) $|AB| = 0 \rightarrow B = O$
(C) $|A^{-1}| = |A|^{-1}$
(D) $|A + A| = 2|A|$

19. If $A=\begin{bmatrix}a&b\\c&d\end{bmatrix}$ (where $bc \neq 0$) satisfies the equations $x^2 + k = 0$, then

(A) $a + d = 0$ (B) $k = -|A|$ (C) $k = |A|$ (D) None of these

20. Let $A=\begin{bmatrix}1&2&2\\2&1&2\\2&2&1\end{bmatrix}$, then

(A) $A^2 - 4A - 5I_3 = O$ (B) $A^{-1} = \frac{1}{5}(A-4I_3)$ (C) A^3 is not invertible (D) A^2 is invertible

21. Let $P=\begin{bmatrix}3&-1&-2\\2&0&\alpha\\3&-5&0\end{bmatrix}$, where $\alpha \in R$, Suppose $Q=[q_{ij}]$ is a matrix such that $PQ = kI$, where $k \in R$, $k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and $\det(Q) = \frac{k^2}{2}$, then –

(A) $\alpha = 0, k = 8$ (B) $4\alpha - k + 8 = 0$ (C) $\det(P\,adj(Q)) = 2^9$ (D) $\det(Q\,adj(P)) = 2^{13}$

22. Let $0<\theta<\frac{\pi}{2}$ and $\Delta(x,\theta)=\begin{vmatrix}x&\tan\theta&\cot\theta\\-\tan\theta&-x&1\\\cot\theta&1&x\end{vmatrix}$ then

(A) $\Delta(0, \theta) = 0$
(B) $\Delta\left(x,\frac{\pi}{4}\right)=x^2+1$
(C) $\text{Min}\,\Delta(1, \theta) = 0$ when $\theta\in\left(0,\frac{\pi}{2}\right)$
(D) $\Delta(x, \theta)$ is independent on x.

23. Select the correct statements for the matrix $A=\begin{bmatrix}-1&3&-5\\2&1&7\\0&6&1\end{bmatrix}$

(A) $|adj(A)| = 625$
(B) $adj\left(\frac{1}{5}A\right)=\frac{1}{25}adj(A)$
(C) $adj(A^{-1}) = -\frac{1}{25}A$
(D) $\begin{bmatrix}-1&2&0\\3&1&6\\-5&7&1\end{bmatrix}^{-1}=\left(A^{-1}\right)^T$

24. Let A and B be two square matrix of order 3, then which of the following statement(s) is/are correct ?

(A) ABA^T is a symmetric matrix

(B) AB – BA is a skew symmetric matrix

(C) If $B = |A|A^{-1}$, $|A| \neq 0$, then $adj(A^T) - B$ is a skew symmetric matrix

(D) If $B + A^T = O$ and A is a skew symmetric matrix, then B^{15} is also skew symmetric matrix

25. Which of the following is (are) NOT the square of a 3×3 matrix with real entries?

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

26. If $A^{10} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$ then

(A) Number of Natural factors of a are 11

(B) b is an integer

(C) Number of Natural factors of $a+b+c+d$ are 22

(D) a + d is a multiple of 13

27. Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ then _____ {Where $A^{-n} = (A^{-1})^n$}

(A) $A^{-n} = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix} \forall n \in N$ (B) $\lim_{n\to\infty} \frac{A^{-n}}{n} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$ (C) $\lim_{n\to\infty} \frac{A^{-n}}{n^2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (D) $\det\left(A^{-n}\right) = 1$

INTEGER OPTION TYPE (0 - 9)

28. Consider, $f(x) = \frac{-x^2}{(x^2-9)(x-7)^2(x-9)(x-3)}$ where a_i are the integral values of x for which $f(x) \geq 0$ and $a_i < a_{i+1} \forall i = 1, 2, \ldots, 8$. If $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$, and $B^3 - pB^2 + qB - rI = O$

Where $B = adj.A$, then find the value of $(2r+p)$.

29. Let $A = \begin{bmatrix} 3 & 2 & 1 \\ 6x^2 & 2x^3 & x^4 \\ 1 & a & a^2 \end{bmatrix}$ then $\frac{d^2}{dx^2}\left(|A(adjA)|^{\frac{1}{3}}\right)$ at $x = a$ is......

30. Let $A = [a_{ij}] (1 \leq i, j \leq 3)$ be a 3×3 matrix and $B = [b_{ij}] (1 \leq i, j \leq 3)$ be a 3×3 matrix such that $b_{ij} = \sum_{k=1}^{3} a_{ik}a_{jk}$. If $\det(A) = 2$, then the value of $\det(B)$ is ________

31. Let (x, y, z) be points with integer co-ordinates satisfying the system of homogeneous equation x + y + z = 0, x + 2y + 3z = 0 and 2x + 3y + 4z = 0, then the number of such points for which $x^2 + y^2 + z^2 \leq 12$.

32. A be a square matrix of order 2 with $|A| \neq 0$ such that $|A + |A| adj(A)| = 0$, then the value of $|A - |A| adj(A)|$ is?

33. Consider the matrices $A = \begin{bmatrix} 2 & 1 \\ 4 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$, then the value of

$tr(A) + tr\left(\frac{ABC}{2}\right) + tr\left(\frac{A(BC)^2}{4}\right) + ...\infty$ is ? {Where tr(A) = sum of diagonal elements}

34. If A is a square matrix of order 3, then find $\left|\left(A - A^T\right)^{105}\right|$.

35. If $3A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{bmatrix}$ and $AA^T = I_3$ then $|x + y|$ is equal to _____

36. For a real number α, if the system $\begin{bmatrix} 1 & \alpha & \alpha^2 \\ \alpha & 1 & \alpha \\ \alpha^2 & \alpha & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ of linear equations, has infinitely many solutions, then

$1 + \alpha + \alpha^2 =$

37. Let the matrix $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$ be a zero divisor of the polynomial $f(x) = x^2 - 4x - 5$. If the sum of all the elements in the matrix A^3 is $3(P)^3$, find the value of P.

38. Total number of distinct $x \in R$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$

39. Let $z = \frac{-1+i\sqrt{3}}{2}$, where $i = \sqrt{-1}$ and r, s ∈ {1, 2, 3}. Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2.
Then the total number of ordered pairs (r, s) for which $P^2 = -I$ is _____

40. Match The column

	COLUMN - I		COLUMN – II
(A)	$(adj\ A)^{-1}$	(P)	$k^{n-1}(adj\ A)$
(B)	$adj\ (A^{-1})$	(Q)	$\frac{A}{\|A\|}$
(C)	adj(kA)	(R)	$\|A\|^{n-2} A$
(D)	adj(adj A)	(S)	$\frac{adj(adj\ A)}{\|A\|^2}$

SUBJECTIVE PROBLEMS

1. Find α, β, γ if $\begin{bmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{bmatrix}$ is orthogonal matrix.

2. Let a system of equation is given as

$$\left.\begin{aligned} 2x+py+6z&=8 \\ x+2y+qz&=5 \\ x+y+3z&=4 \end{aligned}\right\}$$ Then find the value of p & q if the system of equations has

(i) No Solution (ii) Unique Solution (iii) Infinite Many Solution

3. Let there are 40 different elements, then find

(i) Total number of possible orders of matrices which can be formed using these elements.

(ii) Total number of matrices which can be formed using all of these elements.

4. Find the cardinality of set A (i.e. n(A)), where A = {x: x =$[a_{ij}]_{2\times2}$, $a_{ij} \in \{0, 1, 2\}$ & det(x) = 0}. Given, If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then det(A) = ad – bc.

5. Find the number of 2 × 2 matrix satisfying the following conditions

(i) a_{ij} is 1 or –1 (ii) $a_{11}^2 + a_{12}^2 = a_{21}^2 + a_{22}^2 = 2$ (iii) $a_{11}\, a_{21} + a_{12}\, a_{22} = 0$

6. Find the matrix A satisfying the matrix equation, $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \cdot A \cdot \begin{bmatrix} 3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix}$.

7. Find the product of two matrices A & B, where A = $\begin{bmatrix} -5 & 1 & 3 \\ 7 & 1 & -5 \\ 1 & -1 & 1 \end{bmatrix}$ & B = $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and use it to solve the given system of linear equations

$x + y + 2z = 1$, $3x + 2y + z = 7$ and $2x + y + 3z = 2$.

8. Determine the values of a and b for which the system $\begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b \\ 3 \\ -1 \end{bmatrix}$

(i) has a unique solution (ii) has no solution and (iii) has infinitely many solutions

9. If matrix A = $\begin{bmatrix} a & b & c \\ b & c & a \\ c & a & b \end{bmatrix}$ where a, b, c are real numbers such that abc = 1 and $A^TA = I$, then find the value of $a^3 + b^3 + c^3$.

10. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $10\,B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{bmatrix}$. If B is the inverse of A, then find the value of α

11. Find the adjoint of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix}$ and hence, evaluate A^{-1}.

12. For the matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 18 & 2 & 10 \end{bmatrix}$, show that A (adj A) = O.

13. Investigate for what values of λ, μ the simultaneous equations.

    $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have

    (i) no solution (ii) a unique solution (iii) an infinite number of solutions.

14. Let λ and α be real. Find the set of all values of λ for which the system of linear equations

    $\lambda x + \sin\alpha.y + \cos\alpha.z = 0$, $x + \cos\alpha.y + \sin\alpha.z = 0$, $-x + \sin\alpha.y - \cos\alpha.z = 0$

    has a non-trivial solution. For λ = 1, find all values of α.

15. For what value of k does the following system of equations possess a non - trivial solution over the set of rationales: $3x + ky - 2z = 0$, $x + ky + 3z = 0$, $2x+3y - 4z= 0$. For that value of k, find all the solutions of the system.

16. If S is the set of distinct values of 'b' for which the following system of linear equations

    $\left.\begin{array}{l} x+y+z=1 \\ x+ay+z=1 \\ ax+by+z=0 \end{array}\right\}$ has no solution, then S is:

    (A) an empty set (B) An infinite set

    (C) a finite set containing two or more elements (D) A singleton Set

16. D

17. Matrix $A = \begin{bmatrix} x & 3 & 2 \\ 1 & y & 4 \\ 2 & 2 & z \end{bmatrix}$, If xyz = 60 and 8x + 4y + 3z = 20, then A(adj A) is equal to:

    (A) $\begin{bmatrix} 64 & 0 & 0 \\ 0 & 64 & 0 \\ 0 & 0 & 64 \end{bmatrix}$ (B) $\begin{bmatrix} 88 & 0 & 0 \\ 0 & 88 & 0 \\ 0 & 0 & 88 \end{bmatrix}$ (C) $\begin{bmatrix} 68 & 0 & 0 \\ 0 & 68 & 0 \\ 0 & 0 & 68 \end{bmatrix}$ (D) $\begin{bmatrix} 34 & 0 & 0 \\ 0 & 34 & 0 \\ 0 & 0 & 34 \end{bmatrix}$

17. C

18. If n be the number of values of x for which matrix $\Delta(x) = \begin{bmatrix} -x & x & 2 \\ 2 & x & -x \\ x & -2 & -x \end{bmatrix}$ will be singular, then det(Δ(n)) is

    (where det(B) denotes determinant of Matrix B)

    (A) – 8 (B) – 6 (C) 0 (D) 10

18. C

19. A is non-singular square matrix of (n×n) such that $A^2 = -A$, then det (adj (A^n)) is ___

(A) 1, if n is even (B) - 1, if n is odd (C) 1, if n is odd (D) - 1, if n is even

19. A, C

$A^2 = -A \Rightarrow |A|^2 = -|A|$ if n is odd

$\Rightarrow |A| = -1$, A^n adj $A^n = |A^n| I$

$\Rightarrow |A^n|\ |\text{Adj } A^n| = |A^n I|$

$\Rightarrow |\text{Adj } A^n| = 1$

Similarly when n is even $|A| = 1$

$\Rightarrow |\text{Adj } A^n| = 1$

ANSWER KEY & SOLUTION

1. D	2. D	3. B	4. D
5. D	6. D	7. C	8. D
9. B			

10. C

$$\left|\text{adj}\left(\text{adj}\left(\text{adj}\left(\text{adj}A\right)\right)\right)\right| = |A|^{16} = 4^8.5^{16}$$

$$|A| = 10$$

$$x + y + z = 10$$

11: B

$C_1 \leftrightarrow C_2$

$$Z = \begin{vmatrix} 1 & 3+2i & i \\ 3-2i & 2 & 1+i \\ -i & 1-i & 3 \end{vmatrix} \quad \text{Clearly } z = \bar{z}.$$

12: D

Let $\Delta(x) = \begin{vmatrix} x & 3 & x \\ x^2 & x & 6 \\ x & x & 6 \end{vmatrix}$, then $5A + 4B + 3C + 2D + E = \Delta'(1) + \Delta(1)$

Now $\Delta'(1) = -17$ and $\Delta(1) = 0$

13. C

$A^2 = A$ and $B^2 = I$ and $|A| = 0$.

14. A

15. C

Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \\ b_5 & b_6 \end{bmatrix}$ then $\det(BA) = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ 0 & 0 & 0 \end{vmatrix} \times \begin{vmatrix} b_1 & b_2 & 0 \\ b_3 & b_4 & 0 \\ b_5 & b_6 & 0 \end{vmatrix}$

16. A

17. C

$$ABAB = \begin{bmatrix} 8 & 2 & -2 \\ 2 & 5 & 4 \\ -2 & 4 & 5 \end{bmatrix}\begin{bmatrix} 8 & 2 & -2 \\ 2 & 5 & 4 \\ -2 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 72 & 18 & -18 \\ 18 & 45 & 36 \\ -18 & 36 & 45 \end{bmatrix}$$

$$\Rightarrow ABAB = 9AB$$

18. A, C	19. A, C	20. A, B, D	21. B, C

21 B, C

Que 49: $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, $\alpha \in R$ $Q = [q_{ij}]$, $PQ = kI_{3\times3}$, $k \in R - \{0\}$

$q_{23} = -\frac{k}{8}$, $|Q| = \frac{k^2}{2}$.

$|PQ| = |P|\cdot|Q| = k^3 \Rightarrow |P|\cdot\frac{k^2}{2} = k^3$

$\Rightarrow \boxed{|P| = 2k}$ —— (I)

$|P| = \begin{vmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{vmatrix} = 3(5\alpha) + 1(-3\alpha) - 2(-10)$
$= 12\alpha + 20$ —— (II)

so $2k = 12\alpha + 20 \Rightarrow \boxed{k = 6\alpha + 10}$ —— (III)

(A) $\alpha = 0$, $k = 8$ is not satisfying (III) hence its, wrong option.

(C) $\det(P\,\text{adj}(Q)) = |P|\,|\text{adj}\,Q| = |P|\,|Q|^{3-1} = |P|\,|Q|^2$
$= 2k\cdot\left(\frac{k^2}{2}\right)^2 = \frac{k^5}{2}$

Now, $PQ = kI_{3\times3} \Rightarrow P^{-1}PQ = kP^{-1}I_{3\times3}$.
$Q = kP^{-1} = k\cdot\left(\frac{\text{adj}\,P}{|P|}\right)I_{3\times3} = \frac{k\,\text{adj}(P)}{2k}$
$Q = \frac{\text{adj}(P)}{2}$ —— (IV)

$q_{23} = \left(\frac{(\text{adj}(P))_{23}}{2}\right)_{23} = -\frac{1}{2}(-3\alpha + 4) = -\frac{k}{8}$

$\Rightarrow k = 12\alpha + 16$ —— (V)

from (III) & (V) $12\alpha + 16 = 6\alpha + 10 \Rightarrow 6\alpha = -6 \Rightarrow \boxed{\alpha = -1}$ $\boxed{k = 4}$

so $\det(P\,\text{adj}(Q)) = \frac{k^5}{2} = \frac{(2^2)^5}{2} = 2^9$ so option (C) is correct.

Que 49 continue

(B) $4\alpha - k + 8 = 4(-1) - 4 + 8 = 0$
Hence option (B) is also correct.

(D) $\det(Q\,\text{adj}\,P) = |Q|\,|\text{adj}\,P|$
$= |Q|\,|P|^2$.
$= \frac{k^2}{2}(2k)^2 = 2k^4 = 2\cdot(4)^4 = 2^9$.

Hence, options (B & C) are correct.

22. A, C

$\Delta(x,\theta) = x(\tan^2\theta + \cot^2\theta - 1) - x^3$

$\Delta(0,\theta) = 0 \Rightarrow \Delta\left(x, \frac{\pi}{4}\right) = x - x^3$

$\Delta(1,\theta) = \tan^2\theta + \cot^2\theta - 2 \Rightarrow \min \Delta(1,\theta) = 0$

23. A, B, C, D

24. C, D

(A) $(ABA^T)^T = (A^T)^T B^T A^T = AB^T A^T$ Need not be symmetric

(B) $(AB - BA)^T = B^T A^T - A^T B^T$ Need not be skew symmetric

(C) $B = |A| \frac{\text{adj}A}{|A|} \to B = \text{adj } A$, Now, $c = \text{adj}(A^T) - \text{adj}A$

$\therefore c^T = (\text{adj } A^T)^T - (\text{adj}A)^T = \text{adj}A - \text{adj}A^T = -c$

$\therefore$ skew- symmetric matrix.

(D) $A^T = -B$ & $A^T = -A \therefore A = -B$

Now $B^{15} = -A^{15}$. Let $c = B^{15} \therefore C^T = (B^T)^{15} = (-A)^{15} = -A^{15} = B^{15}$

25. B, D

26. A,B, C, D

$A^n = \begin{bmatrix} 2^n & 3^n - 2^n \\ 0 & 3^n \end{bmatrix}$ so $a = 2^{10}$, $a + b + c + d = 2 \times 3^{10}$

Also $a + d = 2^{10} + 3^{10} = 4^5 + 9^5$, is multiple of 13.

27. A,B,C,D

$A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}, A^{-2} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \Rightarrow A^{-n} = \begin{bmatrix} 1 & 0 \\ -n & 1 \end{bmatrix} \Rightarrow |A^{-n}| = 1$

So $\frac{A^{-n}}{n} = \begin{bmatrix} \frac{1}{n} & 0 \\ -1 & \frac{1}{n} \end{bmatrix}$ and $\frac{A^{-n}}{n^2} = \begin{bmatrix} \frac{1}{n^2} & 0 \\ -\frac{1}{n} & \frac{1}{n^2} \end{bmatrix}$

$$\lim_{n\to\infty} \frac{A^{-n}}{n^2} = \begin{bmatrix} \lim\limits_{n\to\infty}\left(\frac{1}{n^2}\right) & \lim\limits_{n\to\infty} 0 \\ \lim\limits_{n\to\infty}\left(-\frac{1}{n}\right) & \lim\limits_{n\to\infty}\left(\frac{1}{n^2}\right) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

28. 13

$$\frac{-x^2}{(x^2-9)(x-7)^2(x-9)(x-3)} \ge 0 \Rightarrow x \in (-3,3) \cup (3,7) \cup (7,9)$$

So $a_i = -2, -1, 0, 1, 2, 4, 5, 6, 8$

$$A = \begin{bmatrix} -2 & -1 & 0 \\ 1 & 2 & 4 \\ 5 & 6 & 8 \end{bmatrix} \Rightarrow B = \begin{bmatrix} -8 & 8 & -4 \\ 12 & -16 & 8 \\ -4 & 7 & -3 \end{bmatrix}$$

Now $p = \text{trace}(B) = -27$ and $r = \det(B) = 16$

29. 0

$\left|A\left(\text{adj } A\right)\right|^{\frac{1}{3}} = |A|$

$\Rightarrow \dfrac{d^2}{dx^2}\left(\left|A\left(\text{adj } A\right)\right|^{\frac{1}{3}}\right) = 0$

30. 4

clearly $B = AA'$

31. 3

32. 4

Let $A = \begin{bmatrix} m & n \\ p & q \end{bmatrix}, \text{adj}(A) = \begin{bmatrix} q & -n \\ -p & m \end{bmatrix} \text{and} |A| = d$

$\left|A + |A| \text{ adj}(A)\right| = \begin{vmatrix} m+qd & n(1-d) \\ p(1-d) & q+md \end{vmatrix} = 0$

$d[(d-1)^2 + (m+q)^2] = 0$

$d = 1, m + q = 0$

now, $\left|A - |A| \text{ adj}(A)\right| = -(m+q)^2 + 4(mq - np) = 4d = 4$

33. 6

$BC = I_2$

$tr(A) + tr\left(\dfrac{ABC}{2}\right) + tr\left(\dfrac{A(BC)^2}{4}\right) + \ldots\ldots\infty = tr(A) + tr\left(\dfrac{A}{2}\right) + tr\left(\dfrac{A}{4}\right) + \ldots\ldots\infty$

$= \dfrac{tr(A)}{1-\frac{1}{2}} = 2tr(A) = 6$

34. 0

as $A - A^T$ is skew-symmetric matrix of order 3.

$\rightarrow (A - A^T)^{105}$ is skew-symmetric of order 2.

$\rightarrow \left|\left(A - A^T\right)^{105}\right| = 0$

35. 3

$x = -2, y = -1 \rightarrow x + y = -3.$

36. 1 37. 38. 2

3 2

Que 51:

$$\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$$

$$\begin{vmatrix} x & x^2 & 1 \\ 2x & 4x^2 & 1 \\ 3x & 9x^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ 2x & 4x^2 & 8x^3 \\ 3x & 9x^2 & 27x^3 \end{vmatrix} = 10$$

$$\Rightarrow x^3 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 3 & 9 & 1 \end{vmatrix} + x^6 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{vmatrix} = 10$$

$$\Rightarrow x^3 \begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & 0 \\ 2 & 8 & 0 \end{vmatrix} + x^6 \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & 4 \\ 3 & 6 & 18 \end{vmatrix} = 10$$

$$\Rightarrow 2x^3 + 12x^6 = 10$$

$$\Rightarrow 6x^6 + x^3 - 5 = 0$$

$$6x^6 + 6x^3 - 5x^3 - 5 = 0$$

$$6x^3(x^3+1) - 5(x^3+1) = 0$$

$$(6x^3-5)(x^3+1) = 0, \quad x \in R \text{ so } x = -1, \left(\tfrac{5}{6}\right)^{1/3}$$

So No of distinct real root x = (2)

39. 1

39 1

Ques 2: $z = \frac{-1+i\sqrt{3}}{2} = \omega$. $\quad r, s = \{1, 2, 3\}$.

$$P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix} \qquad (r,s) \text{ Pairs} = ?$$

$(-z)^r z^{2s} + z^{r+2s}$

$$P^2 = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}\begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix} = \begin{bmatrix} z^{2r} + z^{4s} & \\ z^{2s}(-z)^r + z^{r+2s} & z^{4s}+z^{2r} \end{bmatrix} = -I$$

$\Rightarrow \boxed{z^{2r} + z^{4s} = -1}$ & $\boxed{z^{r+2s} + z^{2s}(-z)^r = 0}$

$\Rightarrow \omega^{2r} + \omega^{4s} + 1 = 0$

$\boxed{\omega^{2r} + \omega^{s} + 1 = 0}$

$z^{r+2s}\left(1 + (-1)^r\right) = 0$

$\Rightarrow r$ must be odd.

$\Rightarrow r = 1$

Ques 2 continue: $r = 1$

So $\omega^2 + \omega^s + 1 = 0$ So $s = 1$ (must

So $(r,s) = (1,1)$ is the only pair.

40. (A) → (B) → (C) → (D) →

SUBJECTIVE PROBLEMS

2. (i) $q = 3, p \neq 2$ (ii) $p \neq 2, q \neq 3$ (iii) $p = 2$

3. (i) 8 (ii) $8 \times 40!$

4. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $a, b, c, d \in \{0, 1, 2\}$. det(A) = ad – bc = 0 → ad = bc.

If $a, b, c, d \in \{0, 1, 2\}$, then product of two such numbers can be 0, 1, 2 or 4.

So let's make cases in which ad = bc.

S. No.	a d = b c	REPRESENTATION	NO. OF SUCH MATRICES
		a d = b c	
		0 0 0 0	
1	0 = 0	0 1 0 1	5 × 5 = 25
		0 2 0 2	
		1 0 1 0	

		2 0	2 0	
2	1 = 1	1 × 1	1 × 1	1
3	2 = 2	1 × 2	1 × 2	2! × 2! = 4
4	4 = 4	2 × 2	2 × 2	1

Hence, total number of elements in set A = 25 + 1 + 4 + 1 = 31.

5. 8

6. $\frac{1}{19}\begin{bmatrix} 48 & -25 \\ -70 & 42 \end{bmatrix}$

7. x = 2, y = 1, z = - 1

8. (i) $a \neq 3, b \in R$ (ii) $a = -3$ and $b \neq 1/3$ (iii) $a = -3, b = 1/3$

9. 2 or 4

10. 5

11. Adj (A) = $\begin{bmatrix} 11 & -9 & 1 \\ -7 & 9 & -2 \\ 2 & -3 & 1 \end{bmatrix}$, $A^{-1} = \begin{bmatrix} \frac{11}{3} & -3 & \frac{1}{3} \\ -\frac{7}{3} & 3 & -\frac{2}{3} \\ \frac{2}{3} & -1 & \frac{1}{3} \end{bmatrix}$

13. (i) $\lambda = 3, \mu \neq 10$ (ii) $\lambda \neq 3, \mu \in R$ (iii) $\lambda = 3, \mu = 10$

14. $2\alpha = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{4}$

15. k = 33/2, $x : y : z = \frac{-15}{2} : 1 : -3$